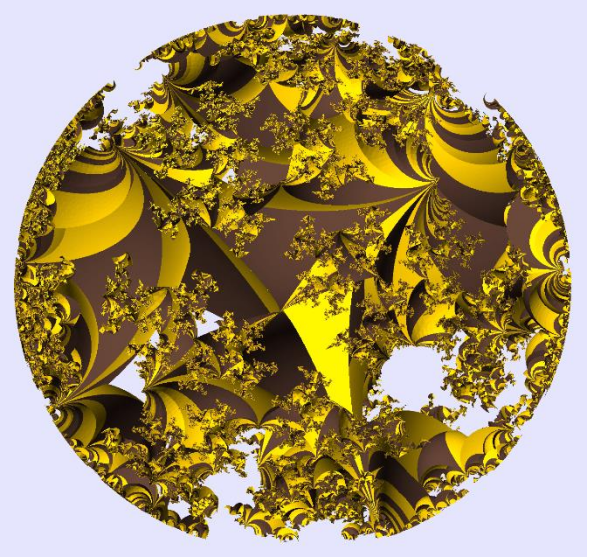
GROUP THEORY



VOL 2

by Dr C. D. H. Cooper

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These notes were prepared for students at Macquarie University in Australia but are freely available to anyone. However if you make use of them and are not a Macquarie University student it would be nice if you could email me at christopherdonaldcooper@gmail.com to let me know where you are from. And, if you are from outside of Australia perhaps you could send me a postcard of where you are from to pin up on my wall (Christopher Cooper, 31 Epping Avenue, EASTWOOD, NSW 2122, Australia).

These notes follow volume 1 and contain some more advanced topics as well as some topics from my own research.

I would like to acknowledge the assistance of Dr Ross Moore, Senior Lecturer at Macquarie University, for pointing out some errors and making some useful suggestions.

Introduction

A central concept in volume 1 was groups described by presentations. This has become the modern way of looking at groups – at least finitely presented ones. Group presentations can be accepted quite readily at an intuitive level, which is the way they were first introduced. However in this volume I present a more rigorous approach via free groups.

In another chapter I discuss soluble groups and, in particular, power-commutator presentations of soluble groups. This is where a finite group is presented with power relations of the form $A^n = I$, for each generator, and [A,B] is a word in the previous generators. For such groups the group table can be constructed in a much more straight-forward way than the Todd-Coxeter algorithm. I also discuss nilpotent groups and their similarity to abelian groups.

Another chapter, on infinite abelian groups, provides a valuable contrast to this computational flavour and provides an excuse for talking about Zorn's Lemma and the Axiom of Choice.

Other chapters include some esoteric topics from my own research. These include power automorphisms, Sylow subgroups of symmetric groups and properties of class equations.

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